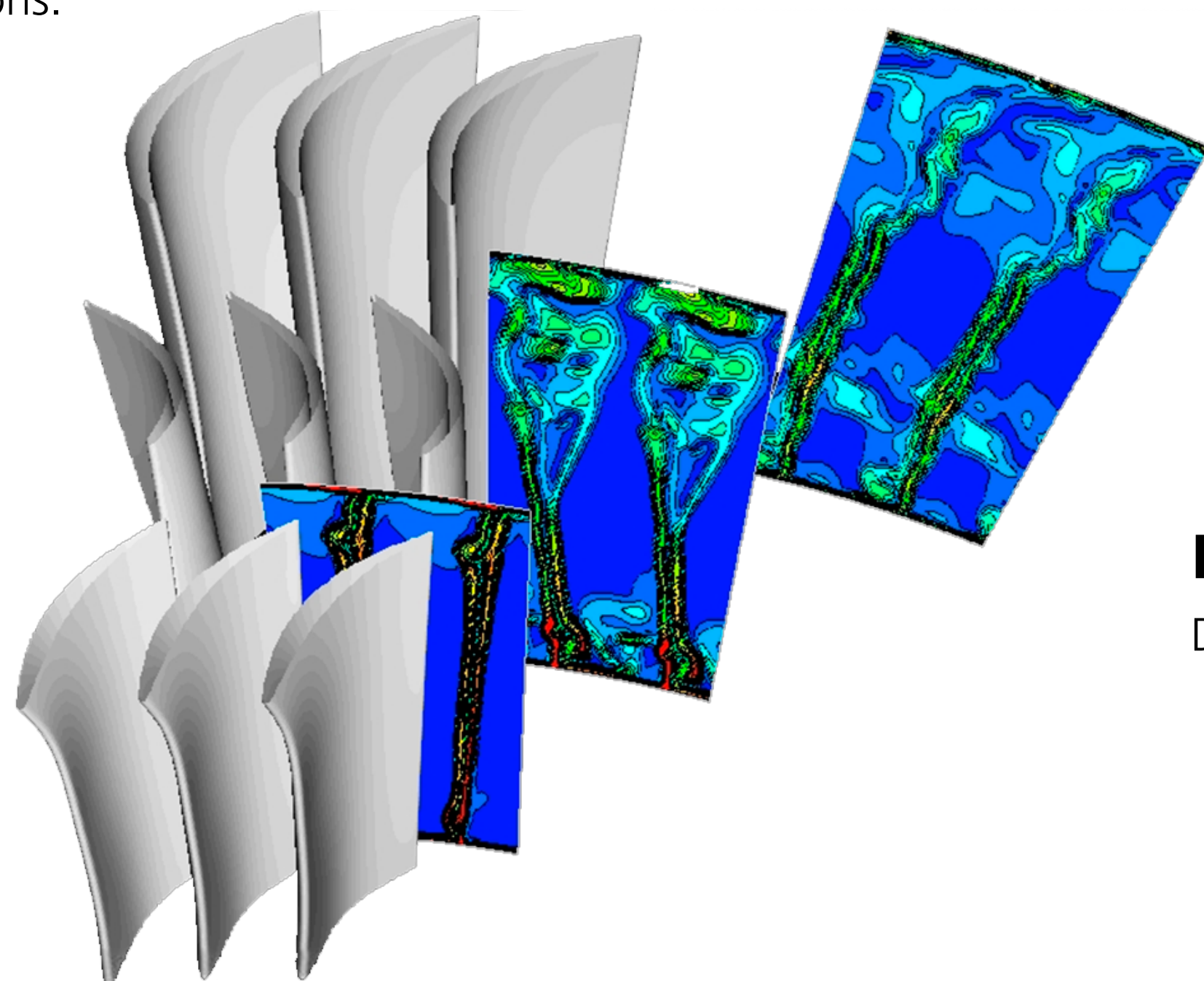




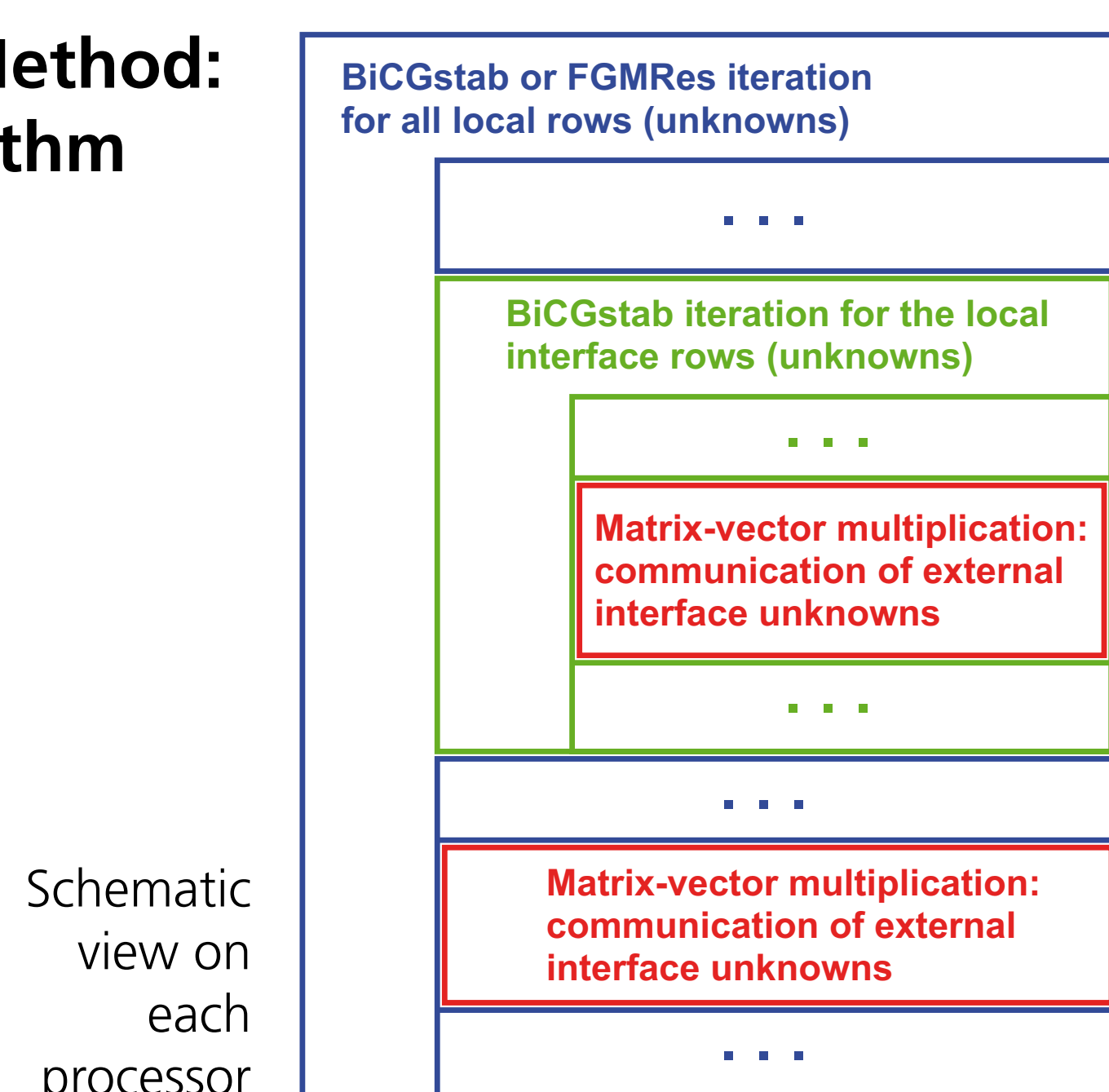
# Parallel Iterative Solvers for Block-Structured CFD Problems

## Parallel Simulation System TRACE

- TRACE: Turbo-machinery Research Aerodynamic Computational Environment
- Developed by the Institute for Propulsion Technology of the German Aerospace Center (DLR-AT)
- Calculates internal turbo-machinery flows
- Finite volume method with block-structured grids
- The linearized TRACE modules require the parallel, iterative solution of large, sparse non-symmetric systems of linear equations.



## DSC Method: Algorithm

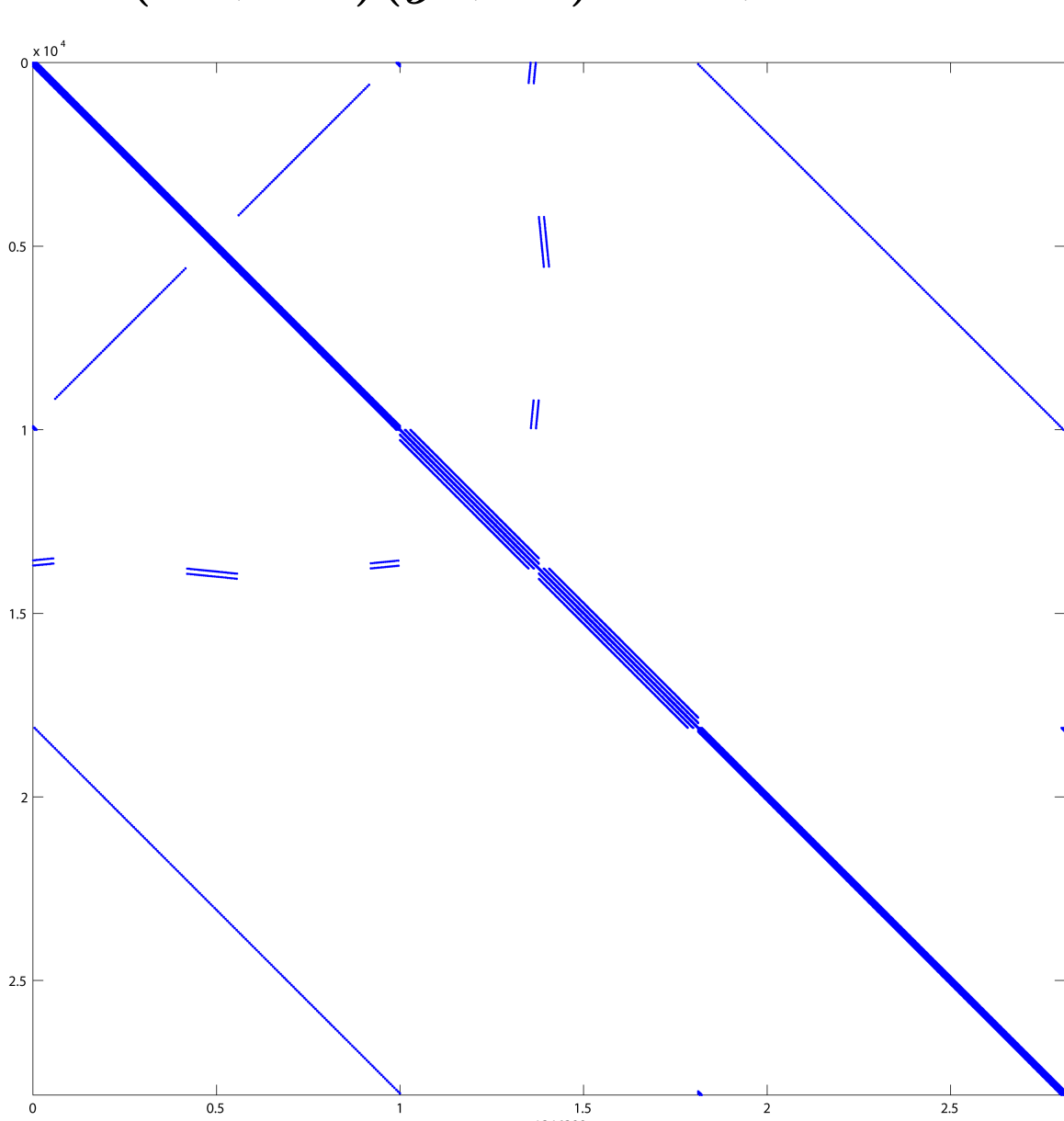


## Typical linearTRACE Matrix Problem

Complex TRACE matrix  
n=28,120; nz=1,246,200; condition:  $6.7 \cdot 10^6$

$$Ax = b$$

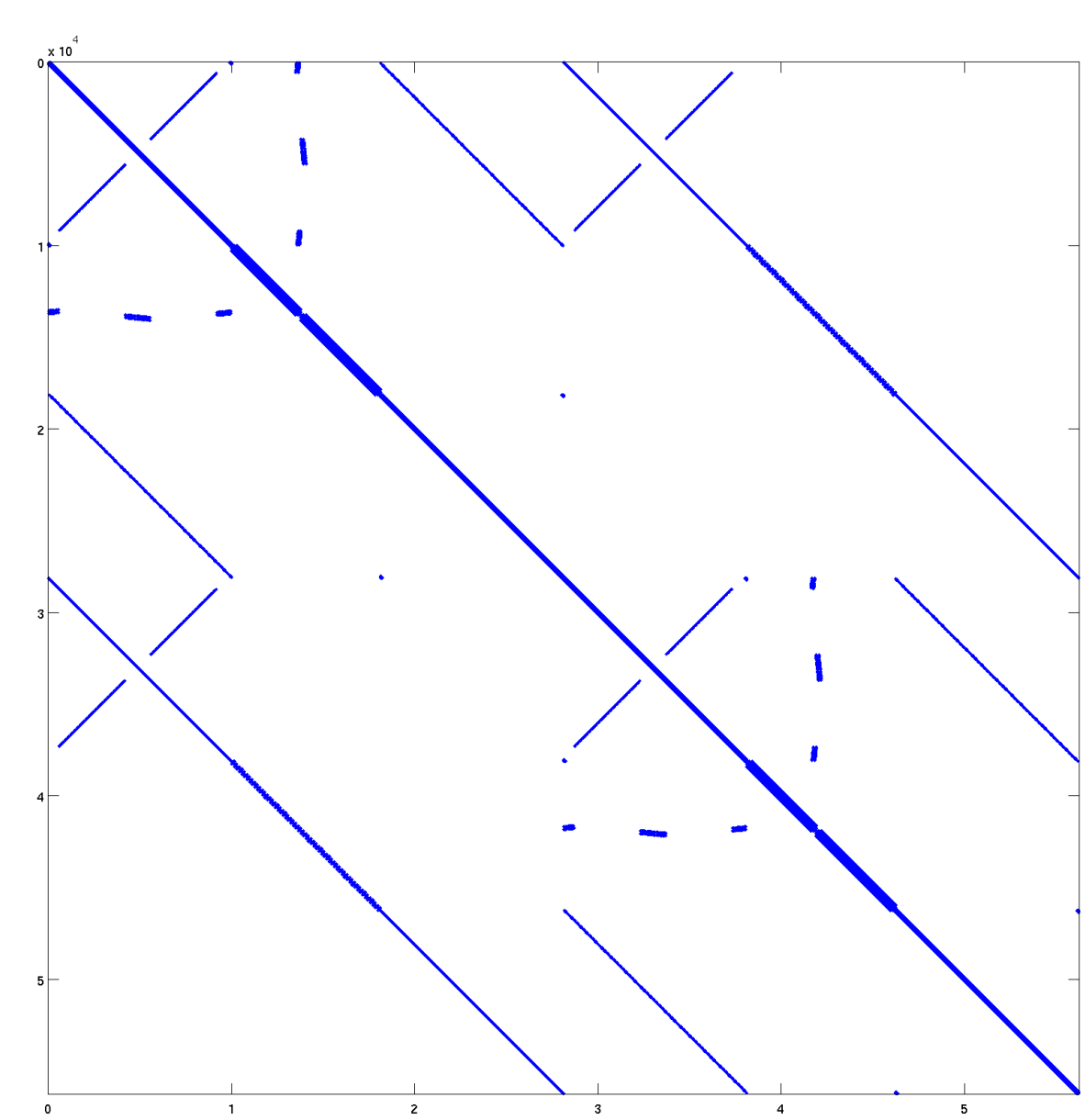
$$\Leftrightarrow (C + iD)(y + iz) = c + id$$



Real TRACE matrix  
n=56,240; nz=2,572,040; condition:  $8.4 \cdot 10^6$

$$\begin{pmatrix} C & -D \\ D & C \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$\Leftrightarrow Gw = e$$



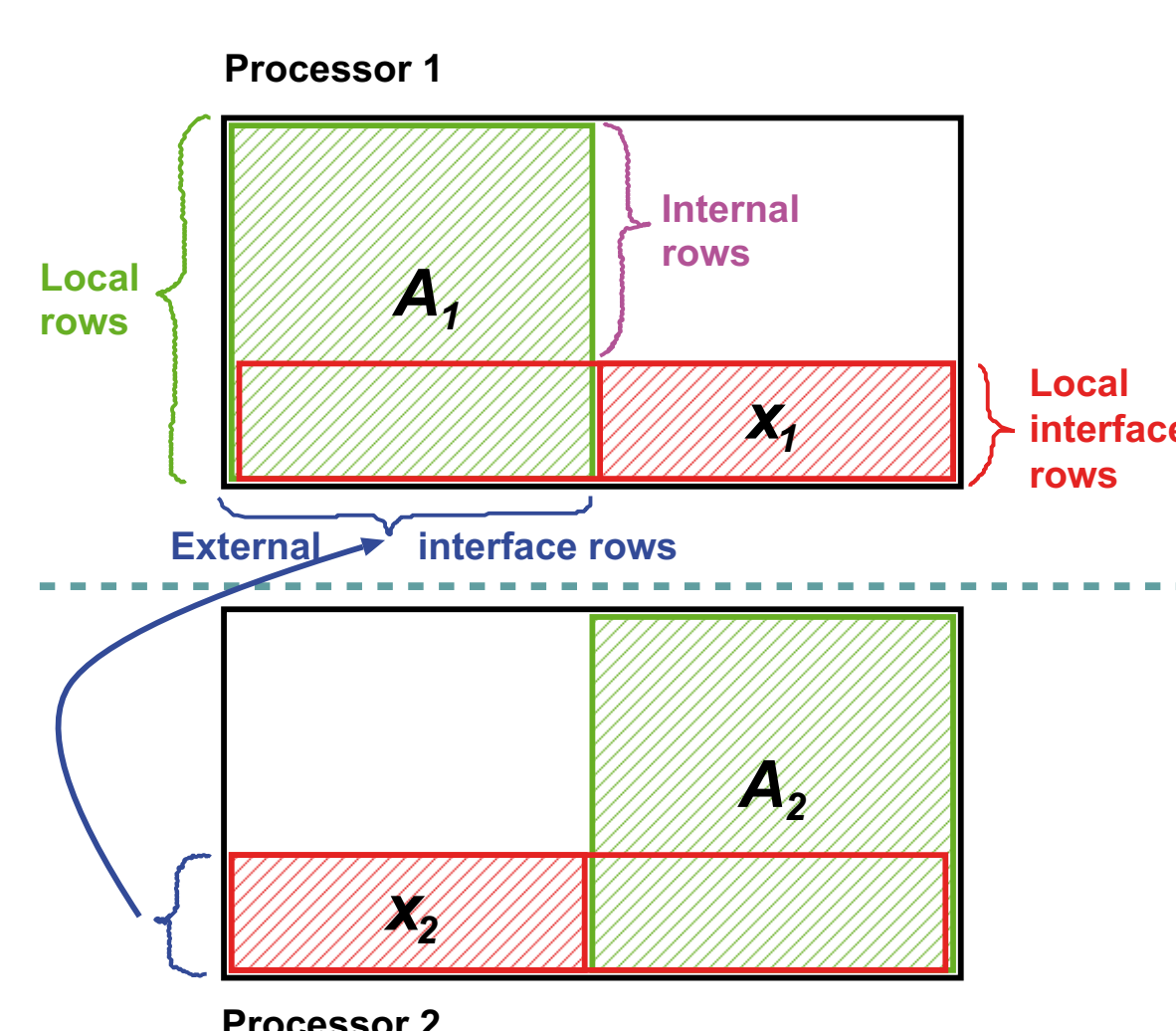
## Preconditioner for TRACE: Background

- Modules *linearTRACE* or *adjointTRACE*  $Ax = b$
- $A$  non-symmetric, complex or real, sparse
- Parallel iterative solver: (F)GMRes with preconditioning  $P^{-1}Ax = P^{-1}b$

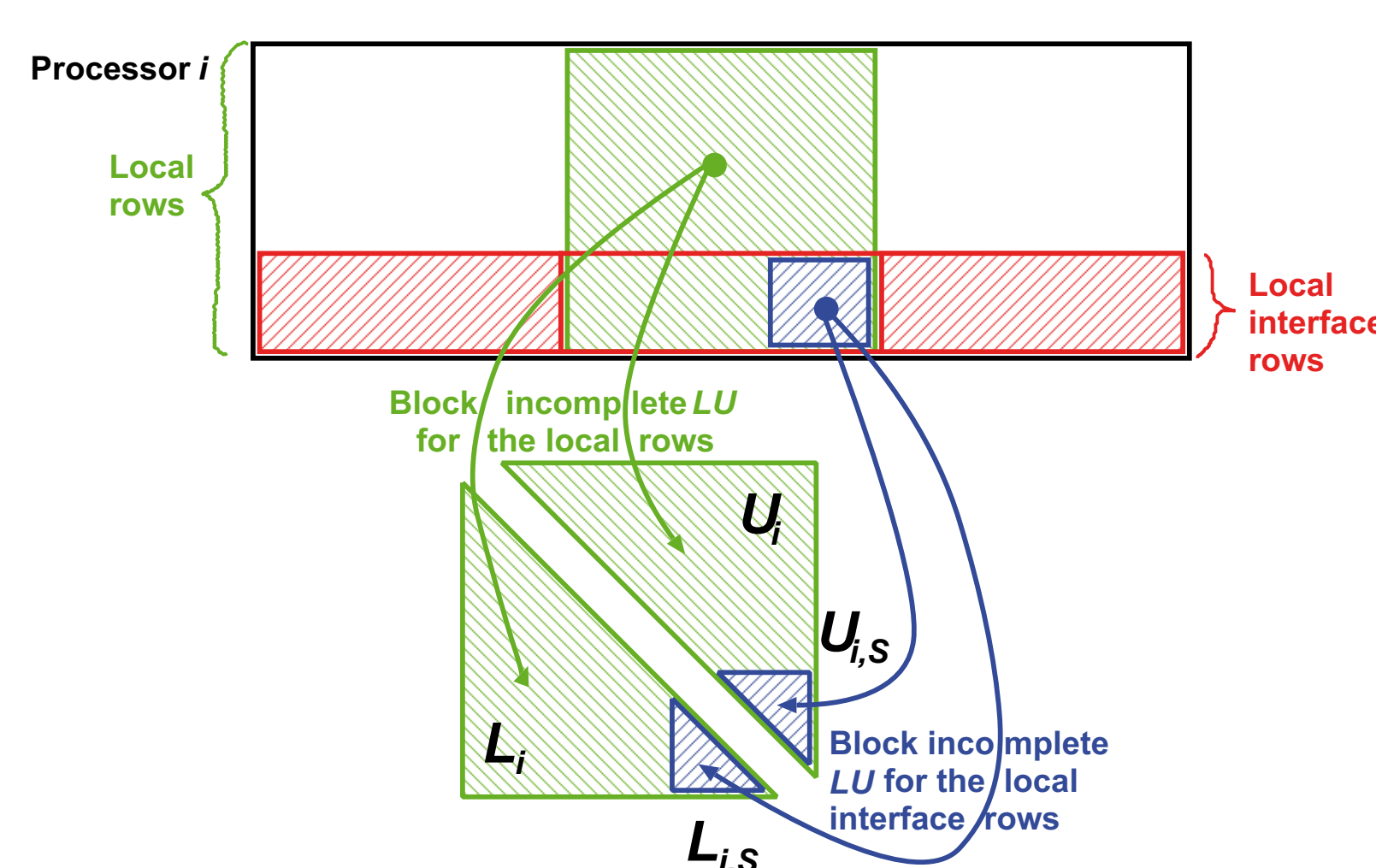
- Distinctly dominates the time behaviour
- Matrix-vector and vector-vector operations
- **Preconditioning usually is the most time-consuming operation**
  - Crucial for scalability
  - **Status:** block-local preconditioning
    - ILU, SSOR
    - **Scalability limited**
- **Goal:** global, scalable preconditioner
  - Experiments with Distributed Schur Complement (DSC) methods

## DSC Method: Definitions

Distributed Matrix, 2 processors

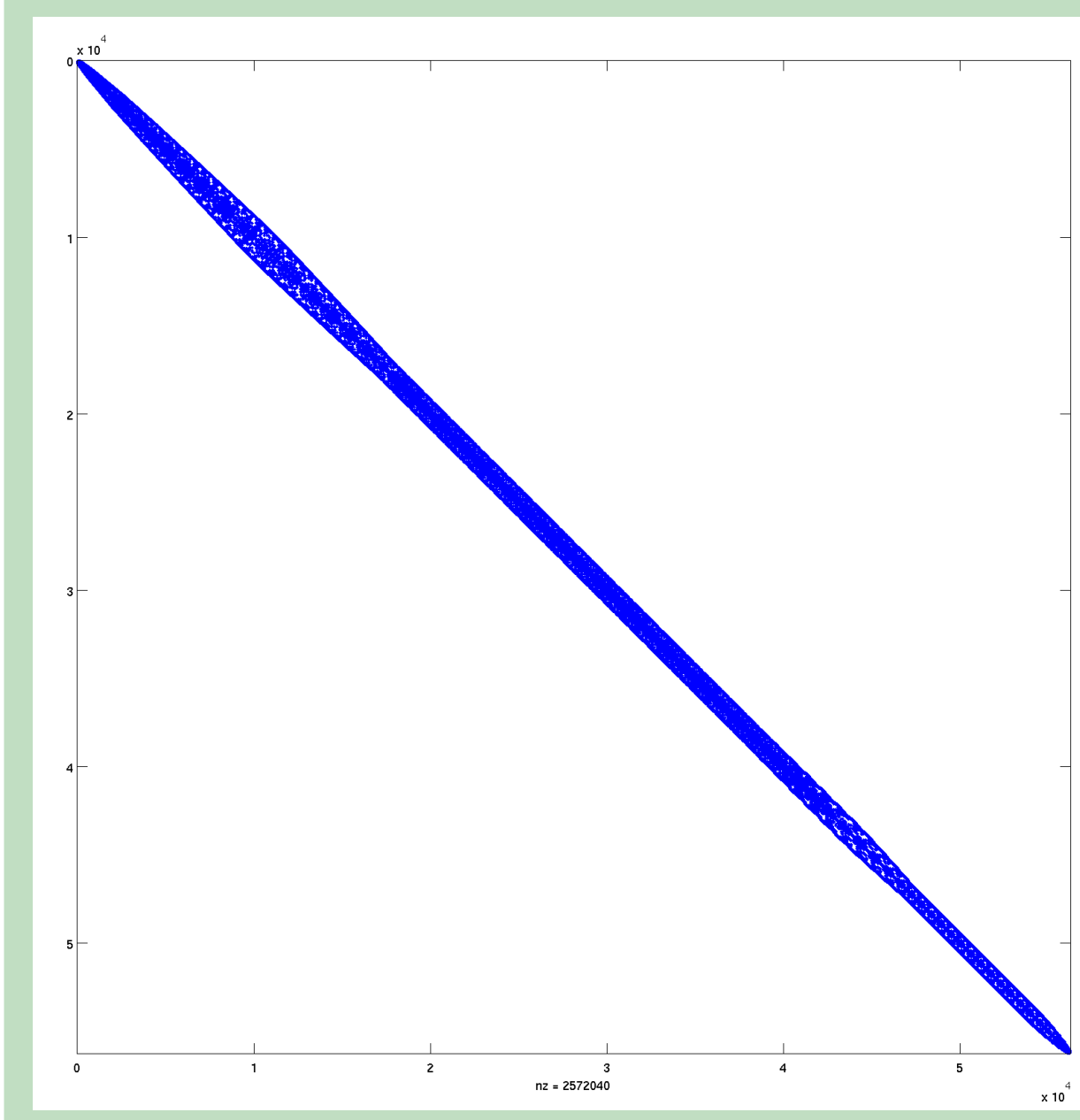


## DSC Method: Incomplete LU Factorizations



## Matrix Permutation for Bandwidth Reduction

After Reverse Cuthill-McKee (RCM)



## Results of the Performance Analysis for TRACE Matrix Problems

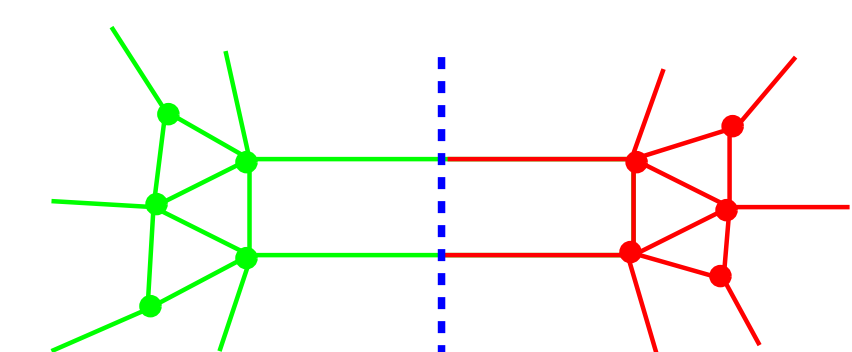
- Block Jacobi preconditioning performs well for small processor counts.
- The DSC method pays off for higher processor counts.
- **Potential method of choice: intelligent solver with**
  - problem and convergence dependent parameter control;
  - problem and convergence dependent preconditioning;
  - preconditioning dependent on the processor count.

## DSC Method and Partitioning

Graph partitioning: *ParMETIS* (University of Minnesota)

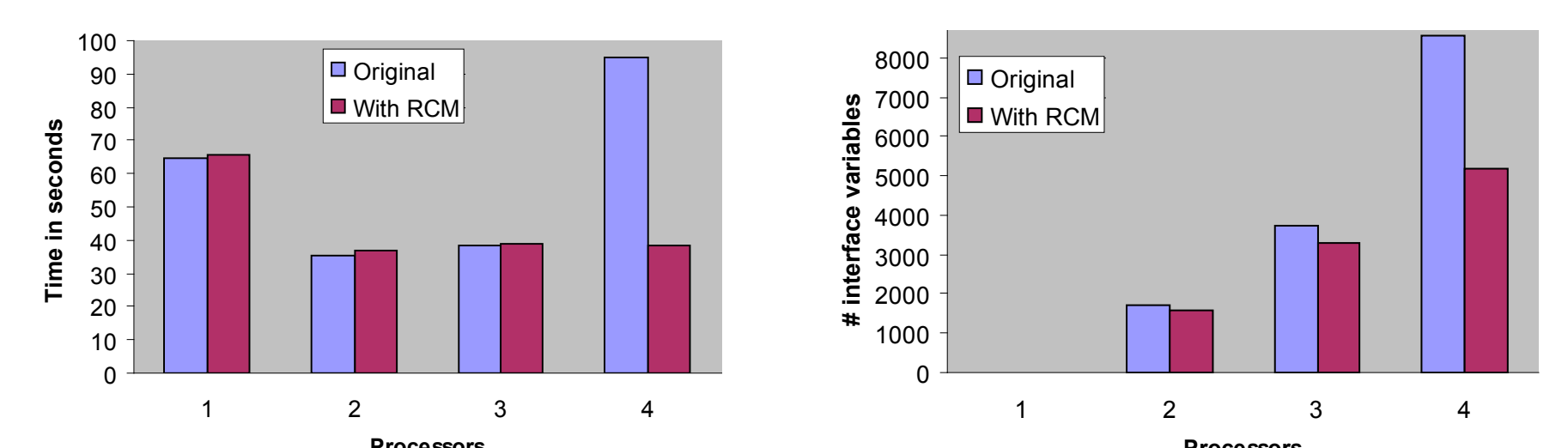
**Goal:**

Minimize the number of edges cut  $\leftrightarrow$  number of interface unknowns



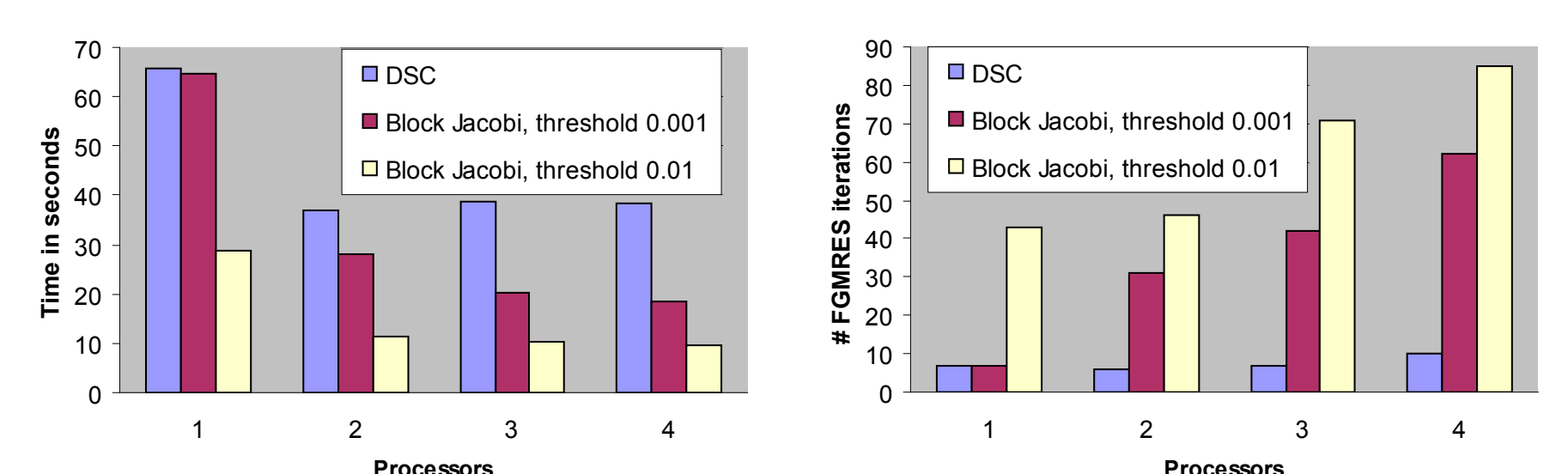
## Performance Tests on an Quad-Core Intel Xeon CPU L5420

Comparison: DSC method for original and RCM permuted matrix



Number of interface variables is significantly lower with RCM.

## Comparison: DSC Method versus Block Jacobi Preconditioning (with RCM)

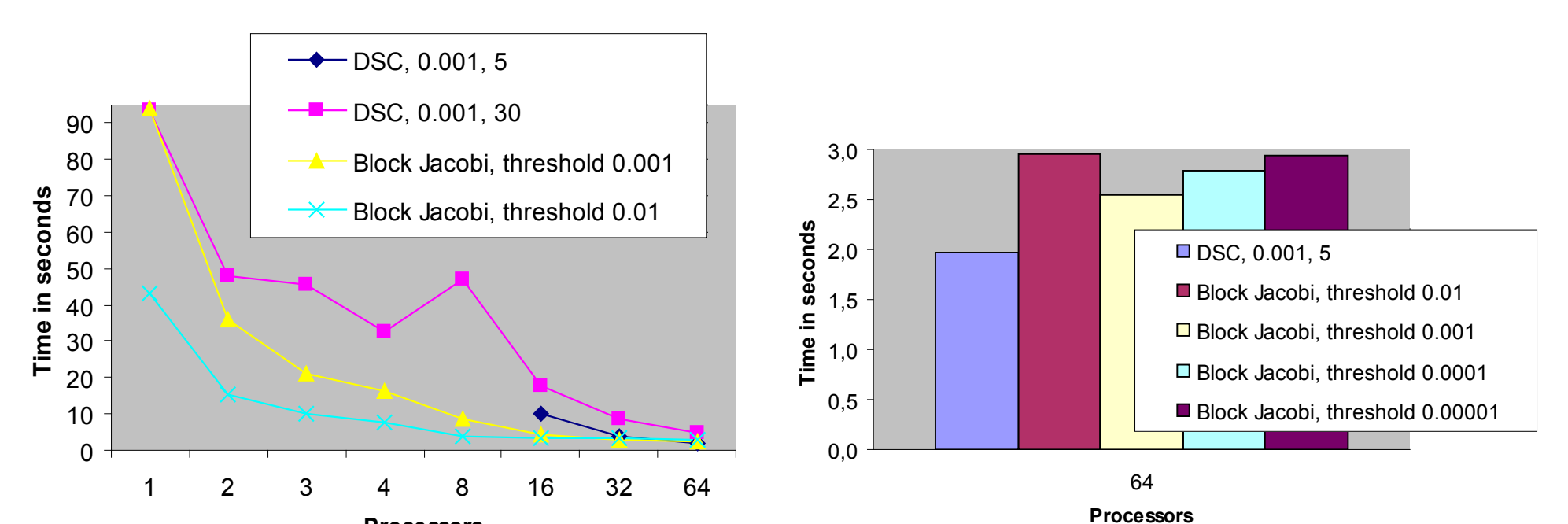


Number of iterations is stable for DSC, but Block Jacobi is faster.

## Performance on a Cluster at DLR-AT

(AMD Opteron Processor 250; Dual-Processor Nodes; 2.4 GHz)

Comparison: DSC method versus Block Jacobi preconditioning (with RCM)



For a high processor count, the DSC method appears to pay off.

